Is it possible to Measure the Weak Phase of a Penguin Diagram?[1]

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Abstract

The $b \to d$ penguin amplitude receives contributions from internal u, c and t-quarks. We show that it is impossible to measure the weak phase of any of these penguin contributions without theoretical input. However, a single assumption involving the hadronic parameters makes it possible to obtain the weak phase and test for the presence of new physics in the $b \to d$ flavour-changing neutral current.

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1 Introduction

One of the most compelling features of CP violation in the B system is that all three interior angles of the unitarity triangle, $\phi_1 (\equiv \beta)$, $\phi_2 (\equiv \alpha)$ and $\phi_3 (\equiv \gamma)$[1], can be measured cleanly, i.e. without theoretical hadronic uncertainties. The B system is thereby expected to provide a test of CP violation in the standard model (SM). Any inconsistency with the predictions of the SM will reveal the much sought after signal of new physics (NP).

NP can affect CP violation in one of two possible ways: through contributions to B decays or to B_d^0-$\overline{B_d^0}$ mixing. Most decay modes of the B-meson are dominated by W-mediated tree-level diagrams and will not be much affected by NP, since in most models of NP there are no contributions that can compete with the SM. Thus, with the exception of penguin-dominated decay modes, NP cannot significantly affect the decays. However, new contributions to B_d^0-$\overline{B_d^0}$ mixing can affect the CP asymmetries[2]. Such NP contributions will affect the extraction of V_{td} and V_{ts}, as well as possible measurements of ϕ_1, ϕ_2 and ϕ_3. Thus, NP enters principally through contributions to B_d^0-$\overline{B_d^0}$ mixing[3].

The angles ϕ_1, ϕ_2 and ϕ_3 are to be measured principally through the modes $B_d^0(t) \to \Psi K_S$, $B_d^0(t) \to \pi\pi$ (or $\rho\pi$)[4], and $B^\pm \to DK^\pm$ (or $D^*K^{*\pm}$)[5], respectively. NP in B_d^0-$\overline{B_d^0}$ mixing will then affect the measurements of ϕ_1 and ϕ_2, but in opposite directions[6]. That is, in the presence of a new-physics phase ϕ_{NP}, the CP angles are changed as follows: $\phi_1 \to \phi_1 - \phi_{NP}$ and $\phi_2 \to \phi_2 + \phi_{NP}$. Hence the sum $\phi_1 + \phi_2 + \phi_3$ is *insensitive* to the NP. However, if ϕ_3 is measured in the decay $B_s^0(t) \to D_s^\pm K^\mp$ [7], then $\phi_1 + \phi_2 + \phi_3 \neq \pi$ can be found if there is NP in B_s^0-$\overline{B_s^0}$ mixing.

The most well known method for detecting NP is to compare the unitary triangle as constructed from measurements of the angles with that constructed from independent measurements of the sides. Any inconsistency will be evidence for new physics. However, since at present the allowed region of the unitarity triangle is rather large, the triangle as constructed from the angles could still lie within the allowed region even if NP is present. Furthermore, even if the ϕ_1-ϕ_2-ϕ_3 triangle lies outside the allowed region, one might still be skeptical about the presence of NP: perhaps the theoretical uncertainties which go into the constraints on the unitarity triangle have been underestimated.

Clearly we would like cleaner, more direct tests of the SM in order to probe for the presence of NP. More promising tests for NP are possible by comparing two distinct decay modes which, in the SM, probe the same CP angle. One can compare the rate asymmetries in $B^\pm \to DK^\pm$ and $B_s^0(t) \to D_s^\pm K^\mp$, both of which measure ϕ_3. A discrepancy between the extracted values would point to NP in B_s^0-$\overline{B_s^0}$ mixing. Similarly, a discrepancy in ϕ_1, as measured via $B_d^0(t) \to \Psi K_S$ and $B_d^0(t) \to \phi K_S$, implies new physics in the $b \to s$ penguin[8]. One can also measure the CP asymmetry in the decay $B_s^0(t) \to \Psi\phi$, which vanishes to a good approximation in the SM. Such an asymmetry would indicate the presence of new physics in B_s^0-$\overline{B_s^0}$ mixing. Note that all such tests probe NP in the $b \to s$ flavour-changing neutral current (FCNC).

One may then ask the question: are there are any direct tests of NP in the $b \to d$ FCNC? For example, consider pure $b \to d$ penguin decays such as $B_d^0 \to K^0\overline{K^0}$ or $B_s^0 \to \phi K_S$, with the asumption that t-quark contribution dominates among up-type quarks in the loop. In such a case the SM would predict that (i) the CP asymmetry in $B_d^0(t) \to K^0\overline{K^0}$ vanishes, and (ii) the CP asymmetry in $B_s^0(t) \to \phi K_S$ measures $\sin 2\phi_1$ [9]. Any discrepancy between measurements

of these CP asymmetries and their predictions would thus imply that there is NP in either B_d^0-$\overline{B_d^0}$ mixing or the $b \to d$ penguin, i.e. in the $b \to d$ FCNC. However, it is well known that $b \to d$ penguins are *not* dominated by the internal t-quark. The contributions of the u- and c-quarks can be as large as 20–50% of that of the t-quark[10]. As a consequence, one cannot probe NP in $b \to d$ FCNC using such modes, and, unfortunately, the answer to the question asked is *no*[11].

2   The CKM Ambiguity

The full $b \to d$ penguin amplitude is a sum of contributions from the three internal up-type quarks in the loop:

$$P = P_u\, V_{ub}^* V_{ud} + P_c\, V_{cb}^* V_{cd} + P_t\, V_{tb}^* V_{td} \; , \tag{1}$$

with $V_{ub} \sim e^{-i\phi_3}$ and $V_{td} \sim e^{-i\phi_1}$. Using the unitarity relation, $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$, the u-quark piece can be eliminated in Eq. (1), allowing us to write

$$P = \mathcal{P}_{cu}\, e^{i\delta_{cu}} + \mathcal{P}_{tu}\, e^{i\delta_{tu}} e^{-i\phi_1} \; , \tag{2}$$

where δ_{cu} and δ_{tu} are strong phases. Now imagine that there were a method in which a series of measurements allowed us to cleanly extract ϕ_1 using the above expression. In this case, we would be able to express $-\phi_1$ as a function of the observables.

On the other hand, we can instead use the unitarity relation to eliminate the t-quark contribution in Eq. (1), yielding

$$P = \mathcal{P}_{ct}\, e^{i\delta_{ct}} + \mathcal{P}_{ut}\, e^{i\delta_{ut}} e^{i\phi_3} \; . \tag{3}$$

Comparing Eqs. (2) and (3), we see that they have the same form. Thus, the same method used to extract $-\phi_1$ from Eq. (2) can be used on Eq. (3) to obtain ϕ_3. That is, we would be able to write ϕ_3 as *the same function* of the observables as was used for $-\phi_1$ above! But this implies that $-\phi_1 = \phi_3$, which clearly does not hold in general.

Due to the ambiguity in the parametrization of the $b \to d$ penguin — which we refer to as the *CKM ambiguity* — we conclude that one cannot cleanly extract the weak phase of any penguin contribution. Indeed, it is *impossible* to cleanly test for the presence of new physics in the $b \to d$ FCNC. Nevertheless, it is instructive to examine in detail a few candidate methods, to see exactly how they fail.

The measurement of the time-dependent rate for the decay $B_d^0(t) \to K^0\overline{K^0}$ can at best allow one to extract the magnitudes and relative phase of $e^{i\phi_1}A$ and $e^{-i\phi_1}\bar{A}$, where A is the amplitude for $B_d^0 \to K^0\overline{K^0}$. With an independent measurement of ϕ_1, there are a total of 4 measurements. Using the form of the $b \to d$ penguin given in Eq. 2, we have $e^{i\phi_1}A = e^{i\phi_1}(\mathcal{P}_{cu}\, e^{i\delta_{cu}} + \mathcal{P}_{tu}\, e^{i\delta_{tu}} e^{-i\phi_1'})$, where we have written the phase ϕ_1' to allow for the possibility of new physics. There are thus 5 theoretical (hadronic) parameters: $\mathcal{P}_{cu}$, $\mathcal{P}_{tu}$, $\delta_{cu}-\delta_{tu}$, ϕ_1, and $\theta_{NP} \equiv \phi_1' - \phi_1$. We see that there are not enough measurements to determine all the theoretical parameters. In fact, there is just one more theoretical unknown than there are measurements. A similar examination[11] of the $B \to \pi\pi$ isospin analysis, Dalitz-plot analysis of $B \to 3\pi$, angular analysis of $B^0 \to VV$ (where V is a vector meson), and a combined isospin + angular analysis of $B \to \rho\rho$ leads to the same conclusion that there is one more unknown than there are measurements.

We thus conclude that, due to the CKM ambiguity, if one wishes to test for the presence of NP in the $b \to d$ FCNC by comparing the weak phase of the t-quark contribution to the $b \to d$ penguin with that of B_d^0-$\overline{B_d^0}$ mixing, it is necessary to make a single assumption[12] about the hadronic parameters.

Acknowledgments

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